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METHOD AND APPARATUS FOR AUTOMATING NEGOTIATIONS BETWEEN **PARTIES**
PROCEDE ET APPAREIL PERMETTANT L'AUTOMATISATION DES NEGOCIATIONS ENTRE
PARTIES

Patent Applicant/Assignee:
OPTIMARK TECHNOLOGIES INC,

Inventor(s):
RICKARD John T,
LUPIEN William A,
WALLACE George A,

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English Abstract

A system that calculates the mutual satisfaction between negotiating **parties** and **maximizes** their mutual satisfaction over a range of decision variables and does so without requiring the **parties** to identify themselves and their positions to each other. A computer (68) accepts a satisfaction function from an offering party who defines his degree of satisfaction to agree to a range of terms upon which the party is desirous of **negotiating** as a function of the relevant decision variables. The **computer** (68) then accepts input from all other **parties** regarding their degree of satisfaction to agree to each of the terms as a function of a particular relevant decision variable. A central matching computer (61) then calculates a joint satisfaction function and calculates the set of decision variables yielding the maximum mutual satisfaction and provides this output to the **parties**

French Abstract

L'invention concerne un systeme qui calcule la satisfaction reciproque de **parties** negociatrices et qui maximise leur satisfaction reciproque sur une plage de variables de decision, et ce sans que les **parties** aient a s'identifier ou a donner leur position respective. Un ordinateur (68) recoit une fonction de satisfaction provenant d'une partie vendeuse qui definit son degre de satisfaction selon lequel il accepte une plage de termes sur la base desquels la partie desire negocier en fonction des variables de decision pertinentes. L'ordinateur (68) recoit ensuite une entree provenant de toutes les autres **parties**, relative a leur degre de satisfaction selon lequel ils acceptent chacun des termes en fonction d'une variable de decision pertinente. Un ordinateur d'adaptation central (61) calcule ensuite une fonction de satisfaction dependant de plusieurs variables et l'ensemble des variables de decision produisant la satisfaction reciproque maximum et envoie ce resultat aux **parties**.

Detailed Description

METHOD AND APPARATUS FOR
AUTOMATING NEGOTIATIONS BETWEEN **PARTIES**
BACKGROUND OF THE INVENTION

The present invention relates to U. S. Application Nos. 08/430,212 and 08/110,666, both of which are hereby incorporated by reference as if repeated herein in their entirety, including the drawings.
The present invention relates generally to methods for performing

negotiations between **parties** and more particularly to a method for automatically performing a negotiation between multiple **parties** without necessarily disclosing the identity and positions of the **parties** to each other. The present invention also relates generally to methods and apparatuses for performing computerized trading, and more particularly to a method and apparatus for performing computerized trading of multiple securities in a single simultaneous transaction. This is known as "linked 5 trading."

In negotiations, **parties** often desire to negotiate a deal involving something of value to both **parties** or multiple **parties**, but are wary of disclosing their willingness to deal on any variable of the deal for fear of losing any negotiating strength. In fact, it is a well-known negotiating tactic to avoid being the first party to suggest an offer, since that sets the stage for the remaining offers. However, it is practically impossible to negotiate without a starting point.

In today's complex world, these negotiations now often involve multiple **parties** and multiple terms. Occasionally, the **parties** do not even know all of the other **parties** to the transaction, e.g., in large exchanges where trades are made between - I millions of **parties** who may never even meet face to face. This presents significant difficulties in reaching an agreement even on a single term, much less multiple terms.

One particularly good example of this is securities trading, and in particular linked trading of securities. Many trading strategies in current markets involve simultaneous purchases and/or sales of multiple securities, where the combined transaction must satisfy a prescribed price **objective**. Simple examples of multiple securities trading include: 1) pairs trading, in which a party is interested in buying one security that is perceived to be undervalued using the proceeds from a simultaneous short sell of a correlated security that is perceived to be overvalued, in expectation that both securities will revert to their nominal values, allowing both positions to be closed out at a profit; and 2) buy-write trades that involve the simultaneous purchase of a stock and sale of a call option on the stock, wherein the call premium received partially hedges the risk of a decline in the value of the stock, at the cost of capping the potential profit of an upside move.

More complex examples of linked trades include extensions of pairs trading to more than two securities. One type of multiple securities trading is known as "basket trading," an example of which is index trading, wherein the **objective** is to buy or sell a defined set of securities (the basket) in a single transaction. Another example is combined equity-currency trades, in which one is purchasing foreign equity in the native currency and simultaneously effecting the required currency exchange trade. Yet another example are "swaps," wherein one income and/or payment stream is swapped for another. More general extensions can be envisioned, involving an arbitrarily complex set of linked trades across multiple securities types.

All current techniques and systems for implementation of linked trades are inefficient and carry some risk for the party desiring the trade for several reasons.

First, the practical requirements of carrying out the multiple trades in significant volume may require hundreds or even thousands of telephone calls and/or keyboard entries into an electronic terminal connected to individual centers of liquidity for the securities involved. Therefore, a common practice among **parties** to such trades is to employ an intermediary who is willing either to take the contra side of the - 2 combined trade, acting as principal, or to negotiate and execute the

simultaneous legs of the combined trade, acting as agent. Obviously, this service is not without cost to the original party, and in the case of a principal trade, the intermediary has now assumed the contra position in the corresponding securities, and in most cases, will want to unwind this position over time. In the case of an agency trade, the realized prices, and even the ability to execute the individual legs of the trade, are uncertain, which exposes the original party to financial risks that cannot be controlled in advance of trade implementation.

Second, the costs incurred in link trading are significantly high, especially where the contra position is negotiated with an intermediary acting as the principal in order to enable the trade to occur with certainty. As an example, an intermediary might charge as much as 3% of the total value of the trade on average, and more if the trade involves illiquid securities, to take the contra position.

Finally, index trading, which is a very prevalent example of multiple security linked trading, often incurs substantial "market impact" costs, in addition to the execution costs, as a result of the leakage of information into the marketplace.

Adverse price moves in the individual securities making up the index can be arranged to the benefit of other market participants (and the detriment of the party desiring the trade) if the other market participants have information about an impending index trade(s). Thus, the linked trader desires both anonymity of the fact that he is trading in linked securities and non-disclosure of his desired position in each of the securities to the market.

The present invention is therefore directed to the **problem** of developing a method and system for automatically negotiating an optimum agreement between **parties** desirous of obtaining such an agreement without necessarily identifying the **parties** and their positions to each other, which method and apparatus is suitable for performing linked trading in a way that **optimizes** the trade prices for the participants on both sides of the trades and does so without disclosing the identity of the trader and his position in the securities at issue.

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SUNB4ARY OF TBE INVENTION

The present invention **solves** the above-identified **problems** by providing a method and system that calculates the mutual satisfaction between the negotiating **parties** and **maximizes** this mutual satisfaction over a range of decision variables and does so without requiring the **parties** to identify themselves and their positions to each other.

According to the present invention for automatically **negotiating** agreements between multiple **parties**, a **computer** accepts a satisfaction profile from an offering party who defines his degree of satisfaction to agree to a range of terms upon which the party is desirous of **negotiating** as a function of the relevant decision variables.

The **computer** then accepts input from all other **parties** regarding their degree of satisfaction to agree to each of the terms as a function of a particular relevant decision variable. The computer then calculates a satisfaction profile for each of these terms based on all of the individual inputs. Next, the computer calculates a joint satisfaction profile for all of the terms as a function of the particular relevant decision variables, and then calculates the mutual satisfaction function for the offering party and the other **parties**, also as a function of the particular relevant decision variables. Finally, the computer calculates the decision variable values yielding the maximum mutual

satisfaction and provides this output to the **parties**.

According to the present invention, a method for automatically trading linked securities includes the steps of receiving a satisfaction function from an offering party who defines its degree of satisfaction to trade multiple securities simultaneously based on the overall cost of the transaction, receiving a satisfaction profile from other traders who indicate their degree of satisfaction to trade particular securities as a function of price/volume, creating a composite satisfaction function for each of the individual securities from the input of all of the individual traders, creating a joint satisfaction function for all of the securities from all of the satisfaction density functions, creating a mutual satisfaction function from the joint satisfaction function and the satisfaction density profile entered by the offering party, **maximizing** the mutual satisfaction function, which establishes a set of prices, volumes and **parties** for trading each of the individual securities, and executing a - 4 trade in the multiple securities simultaneously with the identified **parties** at the prices and volumes established by the maximum mutual satisfaction.

According to the present invention, a system for automatically trading linked securities includes a linked trading workstation at which a linked trader can input a desired linked trade and a satisfaction function that defines its degree of satisfaction to trade multiple securities simultaneously based on the overall cost of the transaction, a plurality of trader workstations at which individual securities traders can enter a satisfaction density profile that indicates their degree of satisfaction to trade particular securities as a function of price/volume, a central control engine determining: (1) a composite satisfaction function for each of the individual securities based on the input of the traders, (2) determining a joint satisfaction function for all of the securities from these satisfaction functions, (3) determining a mutual satisfaction function from the joint satisfaction function and the satisfaction function entered by the linked trader, (4) **maximizing** the mutual satisfaction function, which establishes a set of prices, volumes and **parties** for trading each of the individual securities, and executing simultaneously a trade among the identified **parties** for the multiple securities at the established prices and volumes.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG 1 depicts a satisfaction contour plot of $SL(c)$ for the case of buying one unit of X, and buying two units of X2, respectively.

FIG 2 depicts a satisfaction contour plot of $SL(c)$ for the case of buying one unit of X, and selling two units of X2, respectively.

FIG 3 depicts a satisfaction contour plot of $SL(q)$ for the case of selling one unit of X, and buying two units of X2, respectively.

FIG 4 depicts a satisfaction contour plot of $SL(c)$ for the case of selling one unit of X, and selling two units of X2, respectively.

FIG 5 depicts a contra-side satisfaction contour plot of $S, (P, , P)$ for the case of selling one unit of X, and selling two units of X2, respectively.

FIG 6 depicts a contra-side satisfaction contour plot of $SL(c)$ for the case of selling one unit of X, and buying two units of X2, respectively.

- 5 FIG 7 depicts a contra-side satisfaction contour plot of $S, (P, , P)$ for the case of buying one unit of X, and selling two units of X2, respectively.

FIG 8 depicts a contra-side satisfaction contour plot of $SL(c)$ for the

case of buying one unit of X, and buying two units of X2, respectively.

FIG 9 depicts a contour plot of $MS(P, P)$ for the case of buying one unit of X, and buying two units of X2, respectively.

FIG 10 depicts the segment T, representing the locus of points of tangency between satisfaction contours of $SL(c)$ and $SJPI, P2$.

FIG 11 depicts one embodiment of the system of the present invention for implementing the method of the present invention.

DETAILED DESCRIPTION

The present invention provides a novel method for enabling **parties** to negotiate with each other both quickly and easily without disclosing their willingness to negotiate on specific terms and in fact without necessarily disclosing their identities. In its broadest sense, the present invention is a tool that can discover areas of overlap in negotiating positions between **parties** having a mutual desire to enter into some type of agreement. The applications to which the present invention may be applied are as varied as the imagination of the users. Some examples are disclosed below, however, these are merely indicative of the broad range of possibilities. While the present invention is described mainly herein in the context of securities trading, it is equally applicable to other linked trading applications involving non-securities goods and services, such as commodities, etc. One possible example is as follows.

Airline Negotiations Example

If a party, such as an airline, wishes to negotiate with three unions, such as the airline pilots, the flight attendants and the mechanics unions, the party might employ the present invention. Assuming the airline had a fixed cost (i.e., salaries, benefits, etc.) in mind that it could pay to all employees, and with some constraints, it did not care which unions were paid more than the others, the airline would - 6 employ the present invention as follows. First, the party would determine its satisfaction function based on its understanding of its industry and needs and quantify its desired terms and satisfaction for agreeing to these terms. The method for determining this satisfaction function is different for each industry and participant, and hence is beyond the scope of this invention, but is well understood to those who are responsible for managing companies in these industries. The airline would then enter this data into a computer and transmit that data to the control engine. Each of the three unions would be provided templates for entering their degree of satisfaction for agreeing to the terms, which templates correspond to the arrangement of data of the airline. However, some terms might not necessarily apply to all participants. By enabling the unions and the airline to enter their degree of satisfaction without disclosing it to the other, the present invention permits the **parties** to be more aggressive in specifying terms upon which they might agree. For example, knowing that an earlier agreement might get them more of the total funds available one of the unions might agree to lower wages initially than they might otherwise.

Once all of the data is entered into the system, the system calculates the satisfaction function for each individual term, and then calculates the joint satisfaction function for all of the terms. Finally, the system calculates the mutual satisfaction by determining the area of overlap between the joint satisfaction function and the offering party's satisfaction function. The maximum point of this mutual satisfaction is then determined, which establishes the terms for each of the **parties** and the overall agreement is then specified. The maximum value of the mutual satisfaction may not lie exactly on a discrete term, but may be located between two terms, in which case the nearest term is used. Furthermore, once a point within some tolerable degree is located,

then finding the absolute maximum is not necessary and the search can then end.

Securities Trading Example

A more concrete example can be found in the securities trading arena. U.S.

Patent Application No. 08/430,212, which has been incorporated by reference - 7 above describes the concepts of satisfaction-based representations of mutual trading desire and **optimization** of trading parameters on the basis of mutual satisfaction.

The same concepts are used in the present invention for the individual securities traders.

With regard to the present invention, *inter alia*, this patent application discloses a method that a linked trader uses to enter its orders to buy or sell multiple securities simultaneously in the form of a satisfaction function. The present invention differs from this prior patent application in that it matches the multitude of individual orders with the linked trade order to perform the linked trade and **maximizes** the satisfaction of all traders involved.

Theory

We shall describe the theory behind the present invention in the context of securities trading. The same theory applies equally to the above described situation, and others (such as broadcast time trading, commodities trading, or any agreement where multiple **parties** and/or multiple terms are involved and the **parties** would rather not disclose their willingness to enter into the agreement on certain terms), however, the theory is significantly more abstract in these instances.

Assume a market participant ("linked trader") desires to execute a set of N linked trades involving the simultaneous purchase of V_i units of k different securities, X_i , $i = 1 \dots k$, and sale of V_i units of $N - k$ different securities X_i , where $i = k+1 \dots N$. Any indifferent set of price outcomes from this set of trades can be expressed as a hyperplane in N -dimensional price space.

N

$\sum V_i = C \quad (1)$

where P_i is the price of security X_i , V_i is positive for purchases and negative for sales, and c is the net cost of the trade. Positive values of c represent a net cash outflow from the composite trade, while negative values represent a net cash inflow.

In other words, the linked trader is merely interested in trading certain volumes of securities in a linked trade (i.e., all simultaneously) and does not care about the price obtained for each individual security, but only cares about the total cost of the transaction. Obviously, there is a definite relationship between the costs of the total transaction and the price of an individual security, but the satisfaction function is defined as a function of the cost of the overall transaction rather than the price in the individual securities, as in the prior patent application referenced above.

The linked trader therefore specifies each individual security he desires to trade and a volume associated with that security. Once all of the individual securities are specified, the linked trader then specifies a range of costs for the whole transaction (positive or negative depending whether it is a buy or sell overall) along with a degree of satisfaction for each cost within the range.

For example, Table I is indicative of the type of data entered by the

linked trader. The table indicates the individual securities and the volume of each desired 1 5 to be bought/sold.

Table 1
 Security (X,) Volume (V,)
 EBM 5005000
 Xerox -100,000
 Microsoft 2003000
 Intel 4005000
 AT&T lpopo
 Lucent -5005000
 Viacom lpopo
 Citicorp -200@000
 Conrail -400@000

Continuing the theoretical discussion, the user desiring the linked trade can then define an associated scalar satisfaction function $SL(c)$ for the composite linked trade, which always has the property that $SL(c)$ equals unity for sufficiently small - 9 values of c (which may be negative in the case where net cash inflow is required, as in a sale) and, over some range of c values, transitions monotonically to zero as c increases. This satisfaction function expresses the degree of willingness to perform the prescribed set of linked trades, as a function of the net cost c .

Table 2 indicates the degree of satisfaction the linked trader would have in effecting the overall transaction; ranging from full satisfaction to pay \$3.5 million, to unwillingness to pay \$5.4 million or more.

Table 2
 Overall Cost (\$M) Cost (\$M)
 3.5 1
 3.9 0.9
 4.2 0.75
 4.5 0.5
 4.8 0.25
 5.4 0

Two dimensional examples of $SL(c)$ are shown in the contour plots of FIGS 1-4, wherein the independent variables are P_1 (horizontal axis) and P_2 (vertical axis). These satisfaction profiles cover the four cases of buying X_1 and buying X_2 , buying X_1 and selling X_2 , selling X_1 and buying X_2 , and selling X_1 and selling X_2 , respectively, with each case involving the purchase or sale of one unit of X_1 and two units of X_2 .

In FIG 1, for example, region 5 indicates the region of prices where the trader would be fully satisfied to performed the linked trade. The diagonal lines (i.e., hyperplanes) indicate contour levels of intermediate satisfaction, which decreases approaching the black region. Along any of these diagonal lines, the net cost of the linked trade (and the corresponding satisfaction) is constant, and the trader is therefore indifferent to any price pair lying on a given diagonal line.

Region 1 indicates the region of prices where the trader would be unwilling to perform the linked trade. Moving in the direction of the arrow from region 1 to region 5, one passes through regions of increasing intermediate - 10 satisfaction. In other words, region 2 represents a region of price combinations in which the trader is at least somewhat satisfied to trade. Region 3 represents a region of price combinations where the trader is more satisfied than region 2, but less satisfied than region 4. Region 4 represents a region of price combinations where the trader is more satisfied than region 3 but less than completely satisfied, as in region 5. Finally, region 5 indicates the region of price combinations where the trader would be completely satisfied to trade.

The same is true for FIG 2, however, the whole figure has been rotated

clockwise ninety degrees from FIG 1. In FIG 2, for example, region 10 indicates the region of prices where the trader would be fully satisfied to perform the linked trade. Region 6 indicates the region of prices where the trader would be unwilling to perform the linked trade. Moving in the direction of the arrow from region 6 to region 10, one passes through regions of increasing intermediate satisfaction. In other words, region 7 represents a region of price combinations in which the trader is at least somewhat satisfied to trade. Region 8 represents a region of price combinations where the trader is more satisfied than region 7, but less satisfied than region 9. Region 9 represents a region of price combinations where the trader is more satisfied than region 8 but less than completely satisfied, as in region 10. Finally, region 10 indicates the region of price combinations where the trader would be completely satisfied to trade. As before, along any of these diagonal lines, the net cost of the linked trade (and the corresponding satisfaction) is constant, and therefore the trader is indifferent to any price pair lying on a given line.

As with the previous figure, FIG 3 is similar to FIG 1 but has been rotated ninety degrees counterclockwise from FIG 1. In FIG 3, for example, region 15 indicates the region of prices where the trader would be fully satisfied to perform the linked trade. Region 11 indicates the region of prices where the trader would be unwilling to perform the linked trade. Moving in the direction of the arrow from region 11 to region 15, one passes through regions of increasing intermediate satisfaction. In other words, region 12 represents a region of price combinations in which the trader is at least somewhat satisfied to trade. Region 13 represents a region of price combinations where the trader is more satisfied than region 12, but less satisfied than region 14. Region 14 represents a region of price combinations where the trader is more satisfied than region 13 but less than completely satisfied, as in region 15. Finally, region 15 indicates the region of price combinations where the trader would be completely satisfied to trade. As before, along any of these diagonal lines, the net cost of the linked trade (and the corresponding satisfaction) is constant, and therefore the trader is indifferent to any price pair lying on a given line.

Finally, FIG 4 is similar to FIGS 1-3, but has been rotated ninety degrees counterclockwise from FIG 3. In FIG 4, for example, region 20 indicates the region of prices where the trader would be fully satisfied to perform the linked trade. Region 16 indicates the region of prices where the trader would be unwilling to perform the linked trade. Moving in the direction of the arrow from region 16 to region 20, one passes through regions of increasing intermediate satisfaction. In other words, region 17 represents a region of price combinations in which the trader is at least somewhat satisfied to trade. Region 18 represents a region of price combinations where the trader is more satisfied than region 17, but less satisfied than region 19. Region 19 represents a region of price combinations where the trader is more satisfied than region 18 but less than completely satisfied, as in region 20. Finally, region 20 indicates the region of price combinations where the trader would be completely satisfied to trade. As before, along any of these diagonal lines, the net cost of the linked trade (and the corresponding satisfaction) is constant, and therefore the trader is indifferent to any price pair lying on a given line. This same concept extends to an arbitrary number of dimensions, but cannot be depicted graphically beyond three dimensions.

Having posed the satisfaction function $SL(c)$ for the linked trade, it remains to describe the joint satisfaction function $S(P, V, \dots, V)$, which is a 2N-dimensional function of price and volume in each of the N securities involved in the linked trade. For the moment, let us assume that adequate

contra-side liquidity exists in all of these securities to enable the purchase and/or sale of the desired volume V_i in each security. We shall address the situation when such - 12 liquidity does not exist later.

For security X_i , we can then construct a contra-side satisfaction function $Sc_i(P_i, V)$ by determining, for each price, the volume-weighted contra satisfaction to trade V_i shares at that price. Given our assumption of adequate liquidity, this may be expressed as a scalar function of price alone, $Sc_i(P)$, for the implicit volume V , required for the linked trade.

For example, a sell contra satisfaction function may be constructed by determining the minimum price for which the volume V , can be purchased, along with the corresponding volume-weighted satisfaction at that price, and then incrementing the price upward, calculating volume-weighted satisfaction at each price increment until a volume-weighted satisfaction value of unity is obtained.

Under the assumption of adequate liquidity, the joint contra satisfaction function is then defined as the product of these individual satisfaction functions.

N

SC(P19 0009 PN) $Sc_i(P_i, V)$

This contra-side satisfaction function is defined over a "fuzzy hypercube" in $N+1$ dimensional price space, in the sense that contours of constant satisfaction, which is a fuzzy variable, correspond to nested hypercubes, with rounded corners at intermediate satisfaction values, lying in the positive quadrant of RV. Again employing the two-dimensional cases used in FIGs 1-4 for illustration, we show contour plots of corresponding contra-side satisfaction functions $SL(c)$ in FIGs 58, respectively.

For example, in FIG 5, the region 27 indicates the region of prices where the contra side sellers jointly have unity satisfaction (i.e., are completely satisfied to trade at any pair of prices within this region. The contour lines stepping down to region 22 indicate intermediate levels of joint satisfaction. the traders would be less satisfied to trade at prices in region 26 than region 27, but more satisfied than in region 25. The same is true for regions 24 and 23. Region 22 represents the region where no sellers would be willing to trade.

- 13 In the degenerate case, where all the individual contra side satisfaction functions are step functions that transition abruptly from unity to zero (i.e., corresponding to regular limit orders) the joint contra satisfaction function is unity over the interior of a single hypercube and zero elsewhere. In this case, with reference to FIGS 5-8, there would be an abrupt transition from the black region to farthest most white region without any transition regions.

In FIG 6, the region 28 indicates the region of prices where the contra side buyers/sellers jointly have unity satisfaction (i.e., are completely satisfied) to trade at any pair of prices within this region. The contour lines stepping down to region 33 indicate intermediate levels of joint satisfaction. In other words, the traders . 14 the support regions of the corresponding contra-side satisfaction function $Sc(P_i, V_i)$. When these regions intersect, the hyperplane contours of $SL(c)$ literally slice the corner of the fuzzy hypercube support region of $Sc(P_i, V_i)$.

This feature generalizes to the N -dimensional case as well, although it cannot be visualized beyond three dimensions.

As an illustration, FIG 9 shows the mutual satisfaction support region resulting from the product of $SL(P_i, V_i)$ in FIG 1 and $Sc(P_i, V_i)$ in FIG 5, although only three regions for each are depicted for simplicity sake.

The highest mutual satisfaction occurs in region 55, where the regions of complete satisfaction overlap, i.e., regions 5 (FIG 1) and 27 (FIG 5). Next, region 54 represents a lower value of mutual satisfaction, as it is the intersection of region 27 (FIG 5) with region 4 (FIG 1). Region 52 represents the overlap of regions 26 (FIG 5) and region 5 (FIG 1). As one moves farther away from region 55, the mutual satisfaction decreases. The arrows in FIG 9 represent gradients of decreasing mutual satisfaction. Thus, region 49 has higher mutual satisfaction than region 48 and region 47, etc. The price coordinates of the region 55 indicate the optimum set of prices for this two-security linked trade.

Returning to the theory, the **problem** now reduces to finding the optimum

price point in N-dimensional price space that results in the maximum mutual

satisfaction $MS(P_1, \dots, P_N)$, as defined by

$$MS(P_1, \dots, P_N) = SAC * SC(P_1, \dots, P_N) \quad (3)$$

where c is a function of P_i , V_i , $i = 1 \dots N$ as defined in equation (1).

Optimization Approach

Since none of the V_i in equation (1) are zero, it is straightforward to show that satisfaction contours of $SL(c)$ are not parallel to any axis in N-dimensional price space, and thus are not parallel to any side of the $SC(P_1, \dots, P_N)$ hypercube support region. Thus, the negative gradient of $SL(c)$ with respect to c along any hypercube contour where $0 < SL(c) < 1$ is a vector that points to the interior of the support region of $SC(P_1, \dots, P_N)$, i.e., decreasing values of $SL(c)$ yield increasing (or at least non-decreasing) values of $SC(P_1, \dots, P_N)$.

We first consider the category of cases where the intersection between the support regions of $SL(c)$ and $SC(P_1, \dots, P_N)$ is such that the values of either $SL(c)$ or $SC(P_1, \dots, P_N)$ (or both) are strictly less than unity over the region of intersection, yielding $MS(P_1, \dots, P_N)$ values less than unity in (3). The implication of the gradient geometry is that the optimum mutual satisfaction will occur at a point of tangency between a hyperplane contour of $SL(c)$ and a convex satisfaction contour of $SC(P_1, \dots, P_N)$, since the point of tangency achieves the maximum value of $MS(P_1, \dots, P_N)$ for a given value of $SL(c)$.

Since the individual contra-side satisfaction functions $Sc_j(p)$ in equation (2) are strictly monotonic over their respective regions of transition from unity to zero values, the locus of the points of tangency between hyperplane contours of $SL(c)$ and satisfaction contours of $SC(P_1, \dots, P_N)$ traces out a one-dimensional curvilinear segment T in N-dimensional price space. One end of the segment T is the point of tangency corresponding to the zero satisfaction boundary of $SL(c)$, if $SC(P_1, \dots, P_N) < 1$ at this point, or to the hypercube vertex point corresponding to the unity satisfaction boundary contour of $SC(P_1, \dots, P_N)$ otherwise. The other end of the segment T is the point of tangency corresponding to the unity satisfaction boundary contour of $SL(c)$, if $SC(P_1, \dots, P_N) > 0$ at this point, or to the hypercube vertex point corresponding to the zero satisfaction boundary contour of $SC(P_1, \dots, P_N)$ otherwise.

Along this segment T , the values of $SC(P_1, \dots, P_N)$ decrease strictly monotonically from the former endpoint to the latter, due to the strict monotonicity of the $Sc_j(P)$ functions. Conversely, the values of $SL(c)$ decrease strictly monotonically proceeding from the latter endpoint of T to the former,

P_1, P_2, \dots, P_N

Thus there exists at least one optimum solution (P_1, \dots, P_N) that maximizes $MS(P_1, \dots, P_N)$ at some point along T , which can be determined via a one-dimensional search along the segment T . As an illustration, FIG 10 shows the mutual satisfaction surface resulting from the product of $SL(P_1, P_2)$ in FIG 1 and UP_1, P_2 in FIG 5, although only

three regions for each are depicted for simplicity - 16
 sake. The segment T is shown. The maximum mutual satisfaction occurs
 somewhere on this line segment as discussed above. The price coordinates
 of the peak indicate the optimum set of prices for this two-security
 linked trade.

We now describe a constructive method for evaluating $MS(P_1, \dots, P_N)$
 along the segment T, and thereby determining the optimum set of prices
 that
maximizes $MS(P_1, \dots, P_N)$. Let $(P_i, P_v())$ be the set boundary
 prices
 PCO
 at which the respective individual $Sci(P)$ satisfaction functions just
 attain their zero
 values, and let $P = (P_1, \dots, P_N)$ be the set of boundary prices at which the
 PCI I
 individual $Sci(P)$ satisfaction functions just attain their unity values.
 Let PLO be the point of tangency corresponding to the intersection of the
 zero satisfaction
 boundary hyperplane of $SL(c)$ with a $0 < Sc(P_1, \dots, P_N) < I$ contour of
 $SC(P_1, \dots, P_N)$, where such exists, and let PLI be the point of tangency
 corresponding to the intersection of the unity satisfaction boundary
 hyperplane of $SL(c)$ with a $0 < Sc(P_1, \dots, P_N) < I$ contour of $SC(P_1, \dots, P_N)$,
 where such exists.

1 5 Thus the endpoints of the segment T are given by

0 PO if $SL(PO) < 1$, or TO PLI otherwise; and (4)

--I,

+

+

T i $(P_i) > 0$, or T, PLO otherwise, (5)

1 P, f SL

respectively.

The task now is to evaluate $MS(P_1, \dots, P_N)$ at points along the segment T
 P

to determine the set of prices (P_1, \dots, P_N) that **maximize** this
 quantity. In certain mathematically tractable cases of the functions
 $SL(c)$ and $Sc(P_1, \dots, P_N)$, a rigorous

optimization may be performed using the calculus of variations. In
 other

. 17 instances, a number of standard numerical **optimization**
 approaches may be used.

For the latter cases, interior points of the segment T may be determined
 by finding, for any value of c whose corresponding hyperplane
 satisfaction contour of $SL(c)$ defined by (1) lies between the points To
 and T, the point of tangency $P_1, (c)$ (in N-dimensional price space)
 between that hyperplane contour and its 0 corresponding satisfaction
 contour of $SJPI, \dots, P@d$. As described previously, this is equivalent to
 finding, for the given value of c , the unique set of prices
 (constrained by (1)) that **maximize** $SJPI, \dots, P@d$ and thus

maximize

$MS(P_1, \dots, P_N)$ (the product of $SL(c)$ and $SjPj, \dots, PV$) for that value
 of c . This calculation may be performed for incremental values of c
 lying between those 5 values corresponding to the endpoints of the
 segment T, and the optimum $P_1, (c)$ is selected that results in the maximum
 value of $MS(P_1, \dots, P_N)$.

In practice, satisfaction values of the individual $Sci(P_i)$ functions
 typically are specified at discrete price points (e.g., multiples of
 eighths of a dollar), and each $Sci(P)$ function is assumed to be
 continuous and strictly monotonic with respect to its respective price
 variable P_i between these points. The same holds true for the linked
 trade satisfaction function $SL(c)$. In between these points of

specification of these functions, we assume the satisfaction functions are piecewise linear. For each $Sc_i(P_i)$ let $IP_{j,j} = 0 \dots K_j$ be the set of discrete price points at which $Sc_i(P_i)$ is defined for values of satisfaction S_{ij} between zero and unity, viz.,

So $Wid = S_{ij} \quad (6)$

where

Note that by this definition, if $Sc_i(P_i)$ corresponds to a buyer satisfaction function, then the values of $P_{j,j}$ decrease with increasing values of the index j , and vice versa. If $Sc_i(P_i)$ corresponds to a seller satisfaction function. In this fashion, $Sc_i(P_i)$ always increases strictly monotonically with increasing values of the index j .

Similarly, let $(ck, k = 0 \dots KO)$ be the set of discrete values of net cost at which

$SL(q)$ is defined, viz.,

$SL(ck) = SL'kq$ where $SL_0 = 0$ at c_0 and $SL_K = 1$ at c_{KO} . (8)

Assuming piecewise linearity of the individual satisfaction functions $Sc_j(P)$ and the linked satisfaction function $SL(q)$ between their specified values above, we can then write.

So $(P) = r_i, (P_i - P_i) + S_{ij}$

$P_{iti} < P_i < P_{i+1} \quad (9)$

0 oeo K_i-1

where

$S_{i,j+1} - S_4$

$r_j \quad 0 \dots K_i-1 \quad (10)$

$i'j \quad P_{i,j+1} - P_6,$

and similarly for $SL(q)$.

$SAO = S_k (C - C_d + SLA;$

$C_k < C : @ \quad C_{k+1}$

$k = 0 \dots KO-1$

where

- 19

$SL_{tk+1} - SL_9k$

S

$k' C_{k+1} - C_k \quad (12)$

$k = 0 \dots KO-1$

Outside the price and/or net cost ranges defined above, the satisfaction functions take on unity or zero values, as appropriate.

With the above construction, we can use standard numerical

optimization

techniques to perform an exhaustive evaluation of $MS(P, \dots, P_d)$ over all ck typically involving, at most, a few dozen values) to determine the optimum prices (P_i, \dots, P_d) , which amounts to a very modest computational burden.

In the event of multiple optima along the segment T , generally accepted notions of fair trading practice would give price improvement to the contra-side participants (since the linked trade represents a contingent order and thus has lower priority or standing with respect to non-contingent orders) and thus we - 20

N

$E_{pvi} = C, \quad (13)$

$i=1$

where c , is the maximum value of c in equation (1) for which $SL(c)$ has unity value.

The vertices v , of the hyperplane section bounding the region of unity values of $MS(P_i, \dots, P_n)$ are the points of intersection of (I 3) with the N unity-satisfaction

contour line segments of $Sc(P_i, P_n)$, given by the equations

$P = P_{(1)j}$

$P_i > P_i'$ if $V_2 > 0$, or (14)

1

$0 < P_i < \dots(I)$ if $V_i < 0$; $i = 1 \dots N$

P_i'

The coordinates of the vertices v , thus satisfy the equations

N

$1: P \dots (') V. + P. V_i = C_1, i = 1 \dots N,$

$j=1 \dots i \dots I \dots 1$ (15)

$j \cdot i$

from which we can solve for the P_i coordinate of the i th vertex V_i .

N

$E \ V$

$P_i \ C_1 \ P.$ (16)

V_i

j

Thus the set of vertices of the hyperplane section bounding the region of unity values of $MS(P, \dots, P_N)$ is expressed by.

$V(1) \ 0 \cdot 0 \ FM \ I \ \dots \ N$ (17)

$1 \ P_i \ N$

21 i.e., the i th coordinate of the i th vertex is P_i , as determined by (16) and the j th coordinates of the i th vertex for $j \neq i$ are equal to the coordinates of the

P_i

unity-satisfaction boundary interior vertex of $Sc(P_1, \dots, P_N)$. The optimum

A

centroid point P_c of the hyperplane section is simply the arithmetic average of the vertices in (17), viz.,

N

(18)

$P_c = \dots \ E \ V_i$

$N \ M$

which can be shown by substitution to lie on the hyperplane defined by (14).

Summary

Thus we have constructed a feasible computational procedure for finding the optimum set of prices to execute the linked trade in all cases of intersection between $SL(c)$ and $Sc(P_1, \dots, P_d)$, which **maximizes** the joint mutual satisfaction as defined by (3).

In the above, we assumed adequate liquidity to provide the required volume in each security involved in the linked trade. There may be instances 1-5 where adequate contra volume does not exist in one or more securities. Such cases may be handled by a number of strategies, including.

Determine the minimum percentage of desired volume that is available in any of the involved securities. Adjust all volumes, and the satisfaction functions $SL(c)$ and $Sc(P_1, \dots, P_N)$, to reflect this downsized trade. This option amounts to executing a scaled-down version of the desired trade on a pro-rata basis for each security, preserving the relative volume mix among the individual securities.

Determine the minimum volume available in each involved security, not to exceed the desired volume. Adjust those volumes that are less than the desired volume, and the satisfaction functions $SL(c)$ and $Sc(P_1, \dots, P_N)$, to reflect the reduced trade. This option amounts to an eclectic execution of - 22 the desired trade, preserving as much of the original desired volume in

each security as possible.

Using the above eclectic approach, adjust upward the volume in securities where adequate liquidity is available in order to keep the same $SL(c)$, while adjusting $Sc(P_1, \dots, P_N)$ to reflect the new volumes. This option amounts to executing a trade of similar overall net cost to the original desired trade, but with a different mix of volumes of the requisite securities.

It will be apparent that numerous other strategies may be considered for dealing with cases of inadequate liquidity, all of them falling within the scope of the basic invention.

Entry of the Linked Trade

The party desiring to perform a linked trade can access the system using a specially designated terminal or the same trading terminal with a menu selection for selecting a linked trading profile entry. The linked trading profile includes fields for each security and the desired volume. Once all of the securities have been entered, the screen includes a series of fields for entering the costs for the linked trade and each associated satisfaction. Price is not entered for the individual securities as the costs for the overall linked trade essentially accounts for price in the individual securities.

System

Referring to FIG I 1, the system 60 to implement the present invention includes a central matching controller 61, i.e., a computer, such as an 11BM RS6000/SP, a plurality of trader terminals 62-67 (e.g., PC based workstations), and a linked trader terminal 68. While only one linked trader terminal 68 is shown, the system could employ a large number of these terminals, and in fact, these terminals can be identical to the trader terminals for individual securities markets. During each transaction, however, only one linked trade can be processed at a time. Thus, the linked trades will be processed according to a priority, such as first in time.

- 23

The traders A, through A,, represent traders in one market, traders B, through B,, represent traders in another market, and traders Z, through Zk represent traders in yet another market. A large number of markets could be represented within the system of the present invention, thus enabling transactions in a variety of securities, commodities and equities. For example, traders A, through A,, could represent traders from the Pacific Exchange, while traders B, through B. represent traders in the foreign currency market in London, and traders Z, through Zk could represent traders in the commodities market in Tokyo.

Optionally, each trader terminal could have the capability for performing linked I 0 trades by selecting an option from a menu of choices.

A central database stores all of the information received from the traders for later processing by the control engine. A database extraction routine can be used to extract certain types of information. For example, some traders may enter information regarding specific securities that are not being traded by any other 1 5 traders, hence this data may not be needed for some of the calculations. In this case, the extraction routine may only extract data regarding securities that are part of the linked trade.

This extraction routine operates under control of the main control engine, which ultimately receives all of the data from the plurality of trader terminals and performs the necessary calculations discussed above. Once the identities of all of the parties and the prices and the volumes have all been determined, the control engine then executes the

trade in each of the relevant markets, and informs the **parties** of the results of the trades. Clearing is performed in accordance with known rules and regulations.

In the above system, the workstations can be any type of user terminal, from the low power end to the high power end. For example, the workstation could simply be a dumb terminal that is operated by the central control engine, or a high performance workstation, such as a Sun workstation, or a RISC-based - 24

Claim

I 1. A method for performing simultaneous trading of a plurality of securities, comprising the steps of
a) mapping data from a first party regarding a simultaneous trade of a plurality of securities as a group into a first function expressing degrees of satisfaction of the first party to trade the plurality of securities simultaneously as a group over a desired range of costs;
b) collecting in a central database input from a plurality of traders regarding a degree of satisfaction of each of the plurality of traders to take a position opposite to the first party in at least one of the plurality of securities as a function of price;
I I c) determining from the input of the plurality of traders a joint function expressing a joint degree of satisfaction by one or more second **parties** among the plurality of traders to take an opposite position in all of the plurality of securities as a function of price in each of the plurality of securities;
d) determining a composite function of the first function and the joint function, wherein the composite function represents a mutual degree of satisfaction to execute a trade of the plurality of securities between the first party
1 8 and the one or more of other **parties**; and
e) identifying for each of the plurality of securities a trading price, a trading volume and a trading party among the one or more other **parties** that **maximizes** the composite function.

2 The method according to claim 1, further comprising the step off) executing simultaneously with the first party and the trading **parties** a trade in the plurality of securities at the trading prices and the trading volumes identified in step e).

3 The method according to claim 1, wherein the step b) of collecting further comprises:

- 25

(i) collecting information from the plurality of traders regarding the degrees of satisfaction of the plurality of traders to take either position in the plurality of securities; and
(ii) selecting from the information collected in step b)(i) the input regarding the degrees of satisfaction of the plurality of traders to take a position opposite to the first party in each of the securities.

4 The method according to claim 2, wherein the step b) of collecting further comprises:

(i) collecting information from the plurality of traders regarding the degrees of satisfaction of the plurality of traders to take either position in the plurality of securities; and
(ii) selecting from the information collected in step b)(i) the input regarding the degrees of satisfaction of the plurality of traders to take a position opposite to the first party in each of the securities.

5 The method according to claim 1, wherein the step e) of identifying further comprises:

(i) determining an absolute maximum value of the composite function; and
(ii) defining a region about the absolute maximum value of the composite function; and
(iii) determining a trading price, a trading volume and a trading party for each security of the plurality of securities so that a resulting value of the composite function lies within the region defined in step e) (ii).

6 The method according to claim 5, wherein the step e) (iii) of determining further comprises selecting the trading price, the trading volume and the trading party using predetermined criteria.

7 The method according to claim 5, wherein the predetermined criteria includes a nearest practical trading volume.

8 The method according to claim 5, wherein the predetermined criteria includes a nearest practical trading price.

9 The method according to claim 5, wherein the predetermined criteria includes **maximizing** within said region a trading price in each of the securities in favor of the trading **parties**. I 10. A method for automatically negotiating an agreement between a first party and at least one other party, wherein the agreement includes a plurality of terms, comprising the steps of

a) mapping input from the first party regarding the plurality of terms in the agreement into a first function expressing a degree of satisfaction of the first party

to agree to the plurality of terms over a range of decision variables;

b) collecting input in a central database from at least one other party regarding the degree of satisfaction of the at least one other party to enter into the

agreement with the first party on the plurality of terms;

c) mapping the input into a second function expressing the degree of I I satisfaction of the at least one other party to agree to the terms as a function of

decision variables in each of the terms;

d) determining a composite function of the first function and the second function, wherein the composite function represents a mutual degree of 1 5 satisfaction between the first party and the at least one other party to agree to the plurality of terms; and

e) determining a set of terms and decision variables for the agreement from 1 8 a maximum of the composite function. I I 1. The method according to claim 10, further comprising the step off) executing automatically an agreement according to the set of terms and - 27 decision variables determined in step e).

12 The method according to claim 10, wherein the step b) of collecting further comprises:

(i) collecting information from a plurality of other **parties** regarding

the degrees of satisfaction of the plurality of other **parties** to enter into an

agreement about the plurality of terms; and

(ii) selecting from the information collected in step b) (i) the input regarding the degrees of satisfaction of the at least one other party to enter into the agreement with the first party in each of the plurality of terms in the agreement.

13 The method according to claim I 1, wherein the step b) of collecting further comprises:

(i) collecting information from a plurality of other **parties** regarding

the degrees of satisfaction of the plurality of other **parties** to enter into an agreement about the plurality of terms; and
(ii) selecting from the information collected in step b)(i) the input regarding the degrees of satisfaction of the at least one other party to enter into the agreement with the first party in each of the plurality of terms in the agreement.

14 The method according to claim 10, wherein the step e) of determining further comprises:

- (i) determining an absolute maximum value of the composite function; and
- (ii) defining a region about the absolute maximum value of the composite function; and
- (iii) determining a set of decision variables yielding a value of the composite function that lies within the region defined in step e)(ii). - 28

15 The method according to claim 14, wherein the step e)(iii) of determining further comprises selecting the set of decision variables using predetermined criteria.

16 The method according to claim 14, wherein the predetermined criteria includes a nearest practical decision variable.

17 The method according to claim 14, wherein the predetermined criteria includes within said region **maximizing** the decision variable in favor of the at least one other party.

18 The method according to claim 10, wherein the at least one party includes a plurality of **parties**, and the step e) of determining a set of terms further comprises identifying a set of **parties** agree to the set of terms and decision points.

19 A method for automatically negotiating an agreement between a first party and a plurality of other **parties**, wherein the agreement includes a plurality of terms and the identity of the plurality of other **parties** is not known to the first party or to each other but the **parties** have expressed a willingness to enter into a binding agreement regarding the plurality of terms, comprising the steps of
a) inputting to a computer a first function expressing a degree of willingness of the first party to agree to the plurality of terms over a range of decision variables;

b) collecting input from the plurality of other **parties** including a plurality of second functions expressing a degree of preference by the plurality of other **I I parties** to agree to each of the plurality of terms as a function of the decision variables in each of the terms;

c) determining a joint function of the plurality of second functions as a function of the decision variables in each of the terms;

1 5 d) determining a composite function of the first function and the joint

function, wherein the composite function represents the mutual degree of - 29 preference between the first party and the plurality of other **parties** to agree to the

8 plurality of terms;

e) determining a maximum of the composite function;

f) obtaining a set of terms and decision points for the agreement represented by a point within a predetermined range about the maximum of the

composite function; and

g) executing an agreement which includes the terms obtained in step f).

20 A method for collectively trading a plurality of items, comprising the steps of

- a) collecting data from a first party desirous of simultaneously trading the plurality of items that indicates a volume for each item, a range of costs for the collective trade, and a first factor indicative of the first party's willingness to trade the plurality of items at the specified volumes for each cost within the range of costs, while also collecting data from other **parties** that indicates for a given item, a particular price/volume combination and a second factor indicative of the party's willingness to trade at that particular price/volume combination;
- b) **maximizing** a mutual satisfaction for the first party and the other **parties**, I I wherein the maximum mutual satisfaction solution defines a trade in each of the items between the first party and a subset of the other **parties**;
and
- c) executing automatically a trade defined in step b).

21 The method according to claim 20, further comprising the step of maintaining secret the identities of the first party and the other **parties**, unless a trade is executed, at which time only the **parties** involved in the trade are disclosed.

22 The method according to claim 20, further comprising the step of maintaining secret the first factor and the second factors even after the trade is executed. - 30

23 The method according to claim 20, wherein the plurality of items comprises a plurality of securities.

24 The method according to claim 20, wherein the plurality of items comprises a plurality of non-securities.

25 The method according to claim 20, wherein the plurality of items comprises a plurality of goods.

26 The method according to claim 20, wherein the plurality of items comprises a plurality of services.

27 The method according to claim 20, wherein the first party defines the collective trade by specifying a volume of each item to be traded, and a degree of willingness to trade for each cost within the desired range of costs.

28 The method according to claim 1, wherein the step c) of determining the joint function further comprises:

- (i) constructing a sell contra satisfaction function for each security to be purchased by determining a minimum price for which the volume can be purchased, along with a corresponding volume-weighted satisfaction at that price;
- (ii) incrementing the price upward from the minimum price;
- (iii) calculating the volume-weighted satisfaction at each price increment until a volume-weighted satisfaction value of unity is obtained;
- (iv) constructing a buy contra satisfaction function for each security to be sold by determining a maximum price for which the volume can be sold, along with a corresponding volume-weighted satisfaction at that price;
- (vi) incrementing the price downward from the maximum price;
and

- 31

- (vii) calculating the volume-weighted satisfaction at each price increment until a volume-weighted satisfaction value of unity is

obtained, 18 wherein the joint function is a function of each of the buy and sell contra satisfaction functions.

29 The method according to claim 28, wherein the joint function is a product of the buy and sell contra side satisfaction functions.

30 The method according to claim 28, wherein the buy and sell contra side satisfaction functions include limit orders.

31 The method according to claim 1, further comprising the step of f) defining the composite function by

$$MS(P_{19} \dots P_N) = SL(C) * SC(P_{19} \dots P_N)$$

where c is a function of P_i , V_i , $i = 1$ to N , and $SL(q)$ is the first function and $Sc(P_1, \dots, P_d)$ is the joint function.

32 The method according to claim 31, further comprising the step of g) determining the maximum mutual satisfaction from an intersection between support regions of $SL(q)$ and $Sc(P_1 \dots P_N)$.

33 The method according to claim 32, further comprising the step of h) selecting any point in a particular region of price space over which $MS(P, \dots, P_N)$ is unity to determine the trading price, trading parties and trading volume in which to execute the trade when unity satisfaction regions of $SL(q)$ and $Sc(P_1 \dots P_d)$ intersect, and hence the mutual satisfaction $MS(P_1 \dots P_d)$ is unity over said particular region of price space.

34 The method according to claim 33, further comprising the step of 32 i) selecting the point by selecting a unique centroid point in a hyperplane

section bounding a region of unity values of $MS(P_1, \dots, P_N)$, which hyperplane corresponds to a maximum value of c for which $S, (c)$ has unity value, thereby yielding a most favorable price to each of the trading parties in the region of unity values of $MS(P_1, \dots, P_d)$.

35 The method according to claim 1, further comprising the following steps:

f) determining a minimum percentage of desired volume that is available in any of the involved securities if adequate contra volume does not exist in one or

more securities; and

g) adjusting all volumes, and the first function and the second functions to reflect this downsized trade.

36 The method according to claim 1, further comprising the following steps:

f) executing a scaled-down version of the desired trade on a pro-rata basis for each security and preserving the relative volume mix among the individual securities if adequate contra volume does not exist in one or more securities.

37 The method according to claim 1, further comprising the following steps

f) determining a minimum volume available in each involved security, not to exceed the desired volume if adequate contra volume does not exist in one or

more securities; and

g) adjusting those volumes that are less than the desired volume, and the satisfaction functions $SL(c)$ and $Sc(P_1 \dots P_d)$, to reflect the reduced trade.

38 The method according to claim 1, further comprising the following steps:

f) performing an eclectic execution of the desired trade, preserving as - 33 much of the original desired volume in each security as possible if adequate contra volume does not exist in one or more securities.

39 The method according to claim 1, further comprising the following steps:

f) adjusting upward the volume in securities where adequate liquidity is available in order to keep the same SL(c), while adjusting Sc(PI P") to reflect the new volumes if adequate contra volume does not exist in one or more securities.

40 The method according to claim 1, further comprising the steps off) executing a trade of similar overall net cost to the original desired trade, but with a different mix of volumes of the requisite securities.

41 The method according to claim 1, further comprising the steps off) using a dedicated terminal to enter data when performing a linked trade; g) providing a graphical user interface via which the first party enters data regarding the linked trade, said graphical user interface includes a menu selection for selecting a linked trading profile entry, and the linked trading profile includes fields for each security a desired volume, a plurality of fields for entering the costs for the linked trade and each associated satisfaction.

42 A system for trading a collection of assets and liabilities, comprising: a) a plurality of terminals via which a first party desirous of trading the collection enters data regarding its degree of satisfaction in trading the collection and a plurality of second **parties** enter data regarding their degree of satisfaction in trading components of the collection; and b) a controller executing a trade of the collection in accordance with a **maximization** of a mutual satisfaction of the first and second **parties**.

43 A method for trading a collection of assets and liabilities that includes executing a trade of the collection in accordance with a **maximization** of the - 34 mutual satisfaction of a party desirous of trading the collection and a plurality of other **parties** desirous of taking a contra position in individual components of the collection.

44 A system for automatically trading linked securities comprising: a) a linked trading workstation at which a linked trader can input a satisfaction density profile that defines its degree of satisfaction to trade multiple securities simultaneously based on the overall cost of the transaction; b) a plurality of trader workstations at which individual securities traders can enter a satisfaction density profile that indicates their degree of satisfaction to trade particular securities as a function of price/volume; and c) a central control engine determining: (1) a satisfaction density function for each of the individual securities based on the input of the traders, I 1 (2) determining a joint satisfaction function for all of the securities from the individual satisfaction density functions, (3) determining a mutual satisfaction function from the joint satisfaction function and the satisfaction density profile entered by the linked trader, and (4) **maximizing** the mutual satisfaction function, which establishes a set of prices, volumes and **parties** for trading each of the individual securities, and (5) executing simultaneously a trade among the identified **parties** for the multiple securities at the established prices and volumes.

45 The system according to claim 44, further comprising a central database being coupled to the plurality of trader workstations, the linked trader workstation and the central control engine, and storing input from each of the plurality of traders regarding a degree of

satisfaction of each of the plurality of traders to take a position in a plurality of securities as a function of price. - 35

46 The system according to claim 45, further comprising an extraction routine operating under control of the central control engine and extracting from information stored in the database the degrees of satisfaction of the plurality of traders to take a position opposite to the first party in each of the securities.

47 The system according to claim 44, wherein the plurality of trader workstations include a plurality of personal computers.

48 The system according to claim 44, wherein the plurality of workstations include a plurality of display terminals operated by the central control engine.

49 The system according to claim 44, wherein the plurality of workstations include a high performance workstation.

50 The system according to claim 44, wherein the plurality of workstations include a RISC based workstation.

51 The system according to claim 44, wherein the plurality of workstations include a UNIX based workstation.

52 The system according to claim 44, wherein the central control engine includes a mainframe computer.

53 An apparatus for performing simultaneous trading of a plurality of securities, comprising:

a) a graphical user interface mapping data from a first party regarding a simultaneous trade of a plurality of securities as a group into a first function expressing degrees of satisfaction of the first party to trade the plurality of

securities simultaneously as a group over a desired range of costs;

b) a central database for collecting input from a plurality of traders - 36 regarding a degree of satisfaction of each of the plurality of traders to take a position opposite to the first party in at least one of the plurality of securities as a

function of price; and

c) a processor determining from the input of the plurality of traders a joint function expressing a joint degree of satisfaction by one or more second parties among the plurality of traders to take an opposite position in all of the plurality of securities as a function of price in each of the plurality of securities, determining a composite function of the first function and the joint function, wherein the composite function represents a mutual degree of satisfaction to execute a trade of the plurality of securities between the first party and the one or more of other parties, and identifying for each of the plurality of securities a trading price, a trading volume and a trading party among the one or more other parties that maximizes the composite function.

54 The apparatus according to claim 53, further comprising means for executing simultaneously with the first party and the trading parties a trade in the plurality of securities at the trading prices and the trading volumes identified by the processor.

55 The apparatus according to claim 53, further comprising a routine for extracting information from the database under control of the processor, wherein the database collects and stores information from the plurality of traders regarding the degrees of satisfaction of the plurality of traders to take either position in the plurality of securities, and the extraction routine selects from the information collected by the database the input regarding the degrees of satisfaction of the plurality of traders to take a position opposite to the first party in each of the

securities.

56 A device for automatically negotiating an agreement between a first party and at least one other party, wherein the agreement includes a plurality of terms, comprising:

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a) a user terminal for mapping input from the first party regarding the plurality of terms in the agreement into a first function expressing a degree of satisfaction of the first party to agree to the plurality of terms over a range of decision variables;
b) a plurality of terminals;
c) a database collecting input from at least one other party entered via the plurality of terminals regarding the degree of satisfaction of the at least one other party to enter into the agreement with the first party on the plurality of terms; and
d) a processor mapping the input into a second function expressing the degree of satisfaction of the at least one other party to agree to the terms as a function of decision variables in each of the terms, determining a composite function of the first function and the second function, wherein the composite function represents a mutual degree of satisfaction between the first party and the at least one other party to agree to the plurality of terms, and determining a set of terms and decision variables for the agreement from a maximum of the composite function.

57 The device according to claim 56, further comprising:

e) means for executing automatically an agreement according to the set of terms and decision variables determined by the processor.

58 The device according to claim 56, further comprising a routine for extracting information from the database under control of the processor, wherein the database collects information from a plurality of other **parties** regarding the degrees of satisfaction of the plurality of other **parties** to enter into an agreement about the plurality of terms, and the extraction routine selects from the information collected in the database the input regarding the degrees of satisfaction of the at least one other party to enter into the agreement with the first party in each of the plurality of terms in the agreement.

59 A device for performing simultaneous trading of a plurality of

- 38

securities, comprising:

a) means for mapping data from a first party regarding a simultaneous trade of a plurality of securities as a group into a first function expressing degrees of satisfaction of the first party to trade the plurality of securities simultaneously as a group over a desired range of costs;
b) means for collecting input from a plurality of traders regarding a degree of satisfaction of each of the plurality of traders to take a position opposite to the first party in at least one of the plurality of securities as a function of price; c) means for determining from the input of the plurality of traders a joint function expressing a joint degree of satisfaction by one or more second **parties** among the plurality of traders to take an opposite position in all of the plurality of securities as a function of price in each of the plurality of securities; d) means for determining a composite function of the first function and the joint function, wherein the composite function represents a mutual degree of satisfaction to execute a trade of the plurality of securities between the first party and the one or more of other **parties**; and
e) means for identifying for each of the plurality of securities a trading price, a trading volume and a trading party among the one or more

other parties that maximizes the composite function.

60 The device according to claim 59, further comprising means for executing simultaneously with the first party and the trading parties a trade in the plurality of securities at the trading prices and the trading volumes identified in step e).

61 The device according to claim 59, wherein the means for collecting further comprises:
means for collecting information from the plurality of traders regarding the degrees of satisfaction of the plurality of traders to take either position in the
plurality of securities; and
means for selecting from the information collected in step b) (i) the input - 39 regarding the degrees of satisfaction of the plurality of traders to take a position opposite to the first party in each of the securities.

62 The device according to claim 59, wherein the means for identifying further comprises:
means for determining an absolute maximum value of the composite function;
defining a region about the absolute maximum value of the composite function; and
determining a trading price, a trading volume and a trading party for each security of the plurality of securities so that a resulting value of the composite function lies within the defined region. - 40

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